International Physics Olympiad
«Formula of Unity» / «The Third Millennium»
Year 2024/2025. Qualifying round

Solutions to problems for grade R8

8.1. (7 points) There are two steel cubes placed on each other at the bottom of the aquarium filled with water. Lengths of edges of cubes are $L_1 = 10$ cm and $L_2 = 5$ cm, depth of aquarium is H = 30 cm, density of steel is $\rho = 7900\ kg/m^3$, density of water is $\rho_0 = 1000\ kg/m^3$ Calculate the pressure force of the structure on the bottom of the aquarium when

[1] the first cube lies on the bottom

[2] the second cube lies on the bottom.

Comment. Assume that the acceleration of free fall is 9.8 m/s^2 (*Cherenkov A.A.*)

Answer: 100.

Answer: 90.

Solution. 1)The pressure force of the structure on the bottom of the aquarium is the weight P of the structure , which according to Newton's third law is equal to the reaction force N of the aquarium According to Newton's second law, the resultant force acting on the structure is zero, because the system is in equilibrium:

$$0 = m_1g + m_2g + F_1 + F_2 - N,$$

where F_2 is water pressure on the upper face of the cube with edge L_2, F_1 is water pressure on the part of the upper face of the cube with edge L_1, which is not in contact with the second cube. Therefore:

$$N = m_1g + m_2g + F_1 + F_2$$

Let's calculate:

$$F_1 = p_1S_1 = \rho_0g(H - L_1)(L_1^2 - L_2^2)$$

$F_2 = p_2S_2 = \rho_0g(H - L_1 - L_2)L_2^2$

Then after substitution we obtain that:

$$P = N = (L_1^3 + L_2^3)\rho g + \rho_0g(HL_1^2 - L_1^3 - L_2^3) = 108N$$

2)The pressure force of the structure on the bottom of the aquarium is the weight P of the structure , which according to Newton's third law is equal to the reaction force N of the aquarium

According to Newton's second law, the resultant force acting on the structure is zero, because the system is in equilibrium:

$$0 = m_1g + m_2g + F_1 - F_2 - N,$$

where F_1 is water pressure on the upper face of the cube with edge L_1, F_2 is water pressure on the part of the lower face of the cube with edge L_1, which is not in contact with the second cube. Therefore :

$$N = m_1g + m_2g + F_1 - F_2$$

Let's calculate :

$$m_1g + m_2g = (V_1 + V_2)\rho g = (L_1^3 + L_2^3)\rho g$$

$F_1 = p_1S_1 = \rho_0g(H - L_1 - L_2)L_1^2$

$$F_2 = p_2S_2 = \rho_0g(H - L_2)(L_1^2 - L_2^2)$$

Then after substitution we obtain that

$$P = N = (L_1^3 + L_2^3)\rho g + \rho_0 g(HL_2^2 - L_1^3 - L_2^3) = 85{,}1N$$

Since the minimum number of significant figures in the problem condition is equal to one, we perform corresponding rounding.

8.2. (6 points) Vadim experiments with communicating vessels, which have diameters of $D_1 = 15$ cm and $D_2 = 10$ cm. At first Vadim fills the vessels with water and measures the level of liquid in them. Then he puts a wooden cube of m = 500 g mass into the first vessel and measures the liquid level again. Then Vadim puts the cube into the second vessel and repeats the measurements. By how much will these measurements differ from the original level of water in the vessels

[3] in the first case,

[4] in the second case?

Comment. The density of water is $\rho = 1000\ kg/m^3$. (*Cherenkov A.A.*)

Answer: 0.020 (possible answer: 0.02).

Answer: 0.020 (possible answer: 0.02).

Solution. Since the density of wood is less than the density of water, the cube will float in water. Let us calculate the volume V of the submerged part of the cube. Let's write down Newton's second law for the equilibrium of the cube:

$$F_Arh - mg = 0$$

taking into account that $F_Arh = \rho gV$, we get:

$$V = \frac{m}{\rho}$$

It is clear that when the cube is immersed in the vessels, the water level in them changes by the same value Δh, so that the hydrostatic pressure compensates the pressure exerted by the cube. The cube exerts such pressure as the volume of liquid displaced by it. Thus:

$$p = \frac{F}{S} = \frac{V\rho g}{S_1 + S_2} = \frac{mg}{S_1 + S_2}$$

This pressure is equal to the change in hydrostatic pressure:

$$\frac{mg}{S_1 + S_2} = \rho g \Delta h$$

Therefore

$$\Delta h = \frac{V}{\rho(S_1 + S_2)} = \frac{4m}{\rho\pi(D_1^2 + D_2^2)} \approx 2cm$$

Since the above reasoning is true when the cube is placed in any of the vessels, the liquid level in both cases will change by the same amount.

Since the smallest number of significant figures is two and answer must be written in the SI (units are metres), the answer is 0.020 m.

8.3. (6 points) Cold and warm water are separated by a partition in a calorimeter. The mass of cold water is $m_1 = 200$ g and its temperature is $t_1 = 3^o$ C. The warm water has a mass of $m_2 = 300$ g and its temperature is $t_2 = 10^o$ C. At some point, the partition is removed.

[5] By how many percent will the total volume occupied by the liquids change after the temperatures equalize?

Comment. Assume that the coefficient of thermal expansion is $\alpha = 0.0002\ K^{-1}$. The density of water under normal conditions is $\rho = 1000\ kg/m^3$. (*Cherenkov A.A.*)

Answer: 0.

Solution. 1) From the heat balance equation we can find the equilibrium temperature:

$$cm_1(t - t_1) = cm_2(t_2 - t)$$

Let's transform:

$$m_1t_1 + m_2t_2 = (m_1 + m_2)t$$

2) Let's find the volumes of liquids before and after temperature equalisation. Then the initial volumes of liquids:

$$V_1 = \frac{m_1}{\rho_1} = \frac{m_1}{\rho_0}(1 + \alpha t_1), V_2 = m_2/\rho_2 = m_2/\rho_0(1 + \alpha t_2),$$

where ρ_1,ρ_2 are the initial densities of cold and warm water respectively, ρ_0 - is the density of water under normal conditions. So the total volume of water:

$$V_1 + V_2 = \frac{1}{\rho_0}((m_1 + m_2) + \alpha(m_1t_1 + m_2t_2))$$

Similarly, volumes of liquids after thermal equilibrium is established:

$$\tilde{V_1} = \frac{m_1}{\rho_0}(1 + \alpha t), \qquad \tilde{V_2} = \frac{m_2}{\rho_0}(1 + \alpha t)$$

And their total volume:

$$\tilde{V_1} + \tilde{V_2} = \frac{1}{\rho_0}((m_1 + m_2) + \alpha(m_1 + m_2)t)$$

Substituting $(m_1 + m_2)t = m_1t_1 + m_2t_2$, we get:

$$\tilde{V_1} + \tilde{V_2} = \frac{1}{\rho_0}((m_1 + m_2) + \alpha(m_1t_1 + m_2t_2)) = V_1 + V_2$$

Thus, the volume occupied by the liquids will not change.

8.4. (7 points) The engine of a Lada Vesta car has power of N = 90 horsepower and efficiency of $\eta = 25\%$.

[6] What is the minimum number of liters of petrol to be poured at a petrol station so that the car could drive s = 200 km with a constant speed of v = 72 km/h?

Comment. The density of petrol is $\rho = 0.76\ g/cm^3$, specific heat of combustion of petrol is q = $4.6 * 10^7\ J/kg$. Assume that one horsepower is equal to 735 W. (*Cherenkov A.A.*)

Answer: 76.

Solution. 1) The work done by the engine when the car covers the distance s:

$$A = Fs = Fvt = N\frac{s}{v}$$

2) Let's find the useful work done by the engine:

$$A = \eta Q = \eta qm = \eta q\rho V$$

3) Then we get:

$$N\frac{s}{v} = \eta q\rho V$$

Therefore:

$$V = \frac{Ns}{vq}\rho\eta \approx 75.7l$$

Since the smallest number of significant figures is two, we round and get the answer 76 l.

8.5. (5 points) The equilibrium of solids in liquids is studied in a school laboratory. The teacher took an empty cylindrical glass and carefully immersed it upwards with the bottom and released it. The glass occurred to be in the state of equilibrium.

[7] What is the volume of water poured into the glass?

Comment. The glass has height H = 15 cm, diameter D = 5 cm and mass m = 0.1 kg

(*Cherenkov A.A.*)

Answer: 0.0002.

Solution. 1) The glass is in equilibrium, supported by the Archimedean force of the air in the glass, therefore:

$$mg = \rho V_1 g,$$

where V_1 - volume occupied by the air. Thus:

$$V_1 = \frac{m}{\rho}$$

2) Then the volume of water in the glass:

$$V = V_0 - V_1 = \frac{\pi D^2}{4} H - \frac{m}{\rho} \approx 194.5 cm^3$$

Since the smallest number of significant figures is one and answer must be given in the SI system, then the answer is 0.0002 m^3.

8.6. (7 points) A hollow aluminum ball (outer radius is R = 10 cm, inner radius is r = 9 cm) floats on the surface of water.

[8] What is the maximum density of matter that can be filled inside the ball so that it still floats in the liquid?

Comment. Density of aluminum is $\rho_1 = 2700\ kg/m^3$, density of water is $\rho = 1000\ kg/m^3$.

(*Cherenkov A.A.*)

Answer: 400.

Solution. 1) Let the ball be filled with a matter of maximum density ρ_2. Then the ball will be completely immersed in water, and the Archimedean force is balanced by the total force of gravity:

$$\rho g V = m_1 g + m_2 g$$

$\rho g V = \rho_1 g(V - V_0) + \rho_2 g V_0$

where V_0 and V- internal and external volumes, respectively.

Then we get:

$$\rho_2 = \frac{V\rho - (V - V_0)\rho_1}{V_0} = \frac{V}{V_0}(\rho - \rho_1) + \rho_1 = \frac{R^3}{r^3}(\rho - \rho_1) + \rho_1 \approx 368 kg/m^3$$

Since the smallest number of significant figures is one and answer must be given in the SI system, then the answer is $400 kg/m^3$.

8.7. (7 points) During an extra physics lesson, pupils studied the phenomenon of heat balance by conducting experiments. They placed a piece of ice with a mass of m = 150 g at a temperature of $T_1 = -10^o$ C in a vessel with water, which has a total heat capacity of C = 1550 J/K and temperature of T = 25^o C.

[9] What will be the temperature in the vessel after the system comes to equilibrium?

Comment. Latent heat of fusion of ice is $\lambda = 0.33$ MJ/kg, its specific heat capacity is c = 2.1

$kJ/kg * \text{K}$. (*Cherenkov A.A.*)
Answer: 0.
Solution. 1) Let's find the amount of heat required for ice to melt completely
Heat required to heat the ice:

$$Q_1 = cm\Delta T_1 = 3150J$$

Heat required to completely melt the ice:

$$Q_2 = \lambda m = 49500J$$

Total heat:

$$Q_0 = Q_1 + Q_2 = 52650J$$

2)Let's find the amount of heat required for the calorimeter with water to cool down to zero temperature

$$Q = C\Delta T = 38750J$$

3) Thus, the ice will not melt completely, but will reach its melting tempreture. That is, the final established temperature is zero

8.8. **(6 points)** In a school laboratory, experiments are carried out to demonstrate heat transfer between bodies. There is an aluminum cube heated to a certain temperature. This cube is placed on a piece of ice at a temperature of $t_2 = -20^o$ C.
[10] What must be the minimum temperature of the cube so that it could completely immerse in ice?
Comment. The specific heat capacity of aluminum is $c_1 = 836\ J/kg*\text{K}$, density is $\rho_1 = 2700\ kg/m^3$. Latent heat of fusion of ice is $\lambda = 0.33\ \text{M}J/kg$, its specific heat capacity is $c = 2.1\ kJ/kg * \text{K}$ and density is $\rho_2 = 920\ kg/m^3$. (*Cherenkov A.A.*)
Answer: 150.
Solution. 1) The cube will be completely immersed in ice if its heat is sufficient to heat and melt the amount of ice contained in a volume equal to the volume of the cube. The cube itself will cool down to zero degrees. Then the heat balance equation will be as follows:

$$\lambda m_2 + c_2 m_2(0 - t_2) = c_1 t m_1$$

Where $m_2 = V\rho_2 = a^3\rho_2$ and $m_1 = V\rho_1 = a^3\rho_1$. Therefore:

$$t = \frac{\lambda - c_2 t_2}{c_1}\frac{m_2}{m_1} = \frac{\lambda - ct_2}{c_1}\frac{\rho_2}{\rho_1} \approx 152^o C$$

Since the smallest number of significant figures is two, corresponding rounding is performed.

8.9. **(5 points)** At a children's camp the counsellors decided to teach the children how to build a small electric power station. To do this, they went to the Chernaya River, which forms a waterfall with a height of h = 2 m.
[11] The river flows at a speed of v = 3 m/s, and the cross-section of the stream is s = 3 m^2. How much power can this waterfall develop?
(*Cherenkov A.A.*)
Answer: 200000.
Solution. 1) Flow power is equal to the work that the flow can do per unit time when its kinetic energy at the base of the waterfall is fully converted to work:

$$N = \frac{A}{t} = \frac{E_k}{t}$$

2) According to the law of conservation of energy:

$$E_k = E_{k0} + \Pi = \frac{mv^2}{2} + mgh$$

3) The mass of water passing through the cross-section of the waterfall per unit time:

$$m = V\rho = Svt\rho$$

4) Thus, we find the flow power

$$N = \frac{m}{t}(\frac{v^2}{2} + gh) = Sv\rho(\frac{v^2}{2} + gh) = 216900W$$

Since the smallest number of significant figures is one, answer is rounded.

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Solutions to problems for grade R9

9.1. (6 points) During a practical physics lesson, pupils conduct experiments in a laboratory. They have a calorimeter in which there is a piece of ice with a mass of m = 150 g at temperature of $t_0 = 0^o$ C , as well as a machine that supplies steam at temperature of $t_1 = 100^o$ C into the calorimeter.

[1] What is the minimum mass of vapor to be injected into the calorimeter to obtain water at temperature t = 20^o C?

[2] What will be the mass of obtained water?

Comment. Latent heat of fusion of ice is $\lambda = 0.33$ MJ/kg, latent heat of vaporization of water is l = 2.26 MJ/kg, specific heat capacity of water is c = 4200 $J/kg * K$. Neglect heat capacity of the calorimeter. (*Cherenkov A.A.*)

Answer: 0.174.

Solution.

$$\lambda m + cm(t - t_0) = lm_0 + cm_0(t_1 - t)$$

Therefore
$m_0 = \frac{\lambda + c(t-t_0)}{l+c(t_1-t)} m \approx 24g$ (1)

2) Then the amount of water in the calorimeter is:

$$M = m_0 + m \approx 174g$$

Since the smallest number of significant figures is two, we round off in formula (1). Subsequently we add 2 integers so no further rounding is required. Answer is written in SI units - kilograms.

9.2. (5 points) In a school laboratory, experiments are carried out to demonstrate heat transfer between bodies. There is an aluminum cube heated to a certain temperature. This cube is placed on a piece of ice at a temperature of $t_2 = -20^o$ C.

[3] What must be the minimum temperature of the cube so that it could completely immerse in ice?

Comment. The specific heat capacity of aluminum is $c_1 = 836$ $J/kg*$K, density is $\rho_1 = 2700$ kg/m^3. Latent heat of fusion of ice is $\lambda = 0.33$ MJ/kg, its specific heat capacity is c = 2.1 $kJ/kg *$ K and density is $\rho_2 = 920$ kg/m^3. (*Cherenkov A.A.*)

Answer: 150.

Solution. 1) The cube will be completely immersed in ice if its heat is sufficient to heat and melt the amount of ice contained in a volume equal to the volume of the cube. The cube itself will cool down to zero degrees. Then the heat balance equation will be as follows:

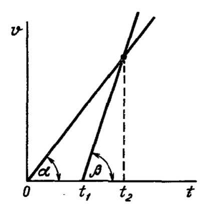
$$\lambda m_2 + c_2 m_2 (0 - t_2) = c_1 t m_1$$

where $m_2 = V\rho_2 = a^3\rho_2$ and $m_1 = V\rho_1 = a^3\rho_1$. Therefore:

$$t = \frac{\lambda - c_2 t_2}{c_1}\frac{m_2}{m_1} = \frac{\lambda - ct_2}{c_1}\frac{\rho_2}{\rho_1} \approx 152^o C$$

Since the smallest number of significant figures is equal to two, we perform the corresponding rounding.

9.3. (5 points) Two toy trains move along the same straight tracks, from the same initial position. Figure shows the graphs of velocities of the trains. It is known that $t_1 = 5$ s, $t_2 = 10$ s.



[4] At what moment of time t_3 will the trains meet?

(*Cherenkov A.A.*)

Answer: 17.

Solution. From the velocity graphs it is clear that the trains are moving with a constant acceleration:

$$a_1 = \operatorname{tg}\alpha = \frac{v_0}{t_2}, a_2 = \operatorname{tg}\beta = \frac{v_0}{t_2 - t_1},$$

where v_0 - velocity of the trains at time t_2. Let us write down the equations of motion of the trains along the axis ox, introduced along the direction of their motion:

$$\begin{cases} x_1(t) = \frac{a_1 t^2}{2} \\ x_2(t) = \frac{a_2 (t-t_1)^2}{2} \end{cases}$$

The trains will meet at a moment t_3, therefore $x_1(t_3) = x_2(t_3)$. So we get:

$$\frac{a_1 t_3^2}{2} = \frac{a_2 (t_3 - t_1)^2}{2}$$

$$\frac{a_1}{a_2} t_3^2 = (t_3 - t_1)^2$$

$$\frac{t_2 - t_1}{t_2} t_3^2 = (t_3 - t_1)^2$$

Solving the quadratic equation with respect to t_3, we obtain that

$$t_3 = t_2 \pm \sqrt{t_2(t_2 - t_1)}$$

Taking into account that t3 > t2, we get the final answer:

$$t_3 = t_2 + \sqrt{t_2(t_2 - t_1)} \approx 17s$$

Since the smallest number of significant figures is equal to one, we round off in the formula when extracting the root. Subsequently we add 2 integers so no further rounding is required.

9.4. (6 points) Petya was gifted a radio-controlled car that can accelerate or decelerate with the same magnitude constant acceleration a = 10 m/s^2 and continues to move uniformly afterwards.

[5] Petya wants to find out what maximum velocity V the car must develop to get from one end of the room to the other in the shortest possible time, provided that it stops at the end of the journey.

Comment. The length of the room is L = 5 m. (*Cherenkov A.A.*)

Answer: 7.

Solution. The time of car movement τ will be the shortest if the average speed of car movement is the greatest, which is obvious The latter, under the conditions of the problem (the beginning and the end of the movement occur with a constant modulus of acceleration), can be only if the car will move with acceleration +a for the first half of the path and with acceleration -a for the second half. Thus, the following relations can be written down:

$$\frac{V\tau}{2 * 2} = \frac{L}{2}, \qquad \frac{\tau}{2} = \frac{V}{a}$$

Therefore

$$V = \sqrt{La} \approx 7m/s$$

Since the smallest number of significant figures is equal to one, we round to integers.

9.5. (8 points) An air balloon descends to the Earth with a constant velocity of u = 2 m/s. At some moment of time a stone with initial velocity of v_0 = 10 m/s relative to the Earth is thrown vertically upwards from this balloon.

[6] What will be the distance L between the balloon and the stone, at the moment when the stone reaches the highest point relative to the Earth?
[7] What is the greatest distance L_{max} the stone will move away from the balloon?
[8] At what time T after the throw will the stone align with the balloon?
Comment. Assume that the acceleration of free fall is 9.8 m/s^2. (*Cherenkov A.A.*)
Answer: 7.
Answer: 7.
Answer: 2.
Solution. 0) Let us introduce a stationary coordinate system connected with the Earth and a moving one rigidly connected with the balloon. Then in the mobile coordinate system the initial velocity of the stone is

$$w_0 = v_0 + u$$

and the laws of motion and change of velocity have the following form:

$$\begin{cases} y(t) = w_0 t - \frac{gt^2}{2} \\ w(t) = w_0 - gt \end{cases}$$

Relative to the Earth, the stone's velocity is:

$$v(t) = v_0 - gt$$

1) When the stone reaches its highest point relative to the Earth, its absolute velocity will be zero: $v(t_1) = 0$. Thus, we find t_1:

$$0 = v_0 - gt_1$$

then

$$t_1 = \frac{v_0}{g}$$

Let's substitute this time into the equation of motion of the stone relative to the balloon and find L, taking into account that $w_0 = v_0 + u$:

$$L = y(t_1) = \frac{v_0}{2g}(v_0 + 2u) \approx 7.1m$$

2) The greatest distance between the stone and the balloon will be at the moment t_2, when the relative velocity of the stone is equal to zero, i.e. $w(t_2) = 0$, thus we find:

$$0 = w_0 - gt_2$$

then

$$t_2 = \frac{w_0}{g}$$

Let's substitute this time into the equation of motion of the stone relative to the balloon and find L, taking into account that $w_0 = v_0 + u$:

$$L_{max} = y(t_2) = \frac{(v_0 + u)^2}{2g} \approx 7.35m$$

3) The condition that the rock will align with the balloon is $y(T) = 0$. Therefore:

$$0 = y(T) = w_0 T - \frac{gT^2}{2}$$

then

$$T = \frac{2(v_0 + u)}{g} \approx 2.45s$$

Since the smallest number of significant figures is equal to one, we round to integers.

9.6. (5 points) During a practical physics class Vanya was studying parallel and series connection of resistors. Unfortunately, he got a set in which two resistors had their nominal value erased. However, Vanya was not confused and connected them first in parallel and then in series to a battery with a voltage of 70 V. In the first case it turned out that the total current flowing was $I_1 = 49$ A, and in the second case - $I_2 = 10$ A.

[9] What are the resistances of the resistors?

Comment. Give your answers separated by semicolons, starting with the smallest.

(*Cherenkov A.A.*)

Answer: 2.0;5.0 (possible answer: 2;5).

Solution. 1) When conductors are connected in parallel, their total resistance is:

$$R_{01} = \frac{R_1 R_2}{R_1 + R_2}$$

Then Ohm's law for a circuit section is:

$$U = I_1 R_0 1$$

2) When conductors are connected in series, their total resistance is:

$$R_0 2 = R_1 + R_2$$

Then Ohm's law for a circuit section is:

$$U = I_2 R_0 2$$

3) Thus we obtain a system of equations for determining R_1,R_2:

$$\begin{cases} \frac{U}{I_1} = \frac{R_1 R_2}{R_1 + R_2} \\ \frac{U}{I_2} = R_1 + R_2 \end{cases}$$

Taking into account that the smallest number of significant figures is two, the correct answer is:

$$R_1 = 2.0\Omega, \quad R_2 = 5.0\Omega$$

9.7. (6 points) A hollow aluminum ball (outer radius is R = 10cm, inner radius is r = 9cm) floats on the surface of water.

[10] What is the maximum density of matter that can be filled inside the ball so that it still floats in the liquid?

Comment. Density of aluminum is $\rho_1 = 2700\ kg/m^3$, density of water is $\rho = 1000\ kg/m^3$.

(*Cherenkov A.A.*)

Answer: 400.

Solution. 1) Let the ball be filled with a matter of maximum density ρ_2. Then the ball will be completely immersed in water, and the Archimedean force is balanced by the total force of gravity:

$$\rho g V = m_1 g + m_2 g$$

$$\rho g V = \rho_1 g (V - V_0) + \rho_2 g V_0$$

where V_0 and V- internal and external volumes, respectively.

Then we get:

$$\rho_2 = \frac{V\rho - (V - V_0)\rho_1}{V_0} = \frac{V}{V_0}(\rho - \rho_1) + \rho_1 = \frac{R^3}{r^3}(\rho - \rho_1) + \rho_1 \approx 368kg/m^3$$

Since the smallest number of significant figures is one and answer must be given in SI system, then the correct answer is $400kg/m^3$.

9.8. (6 points) In a physics class at school electric circuits are studied. A resistor of R = 4 Ohm was connected to a source with internal resistance of r = 2 Ohm, having EMF of 10 V. Then a second resistor of the same value was connected. At first it was connected in parallel and then in series.

[11] Find the ratio of powers released on the first resistor in the first and second cases.

(*Cherenkov A.A.*)

Answer: 1.6 (possible answer: 2).

Solution. 1) Let's consider the parallel connection of the second conductor. Then the total external resistance:

$$R_0 = \frac{R^2}{2R} = \frac{R}{2} = r$$

According to Ohm's law:

$$I = \frac{\varepsilon}{R_0 + r} = \frac{\varepsilon}{2r}$$

Since the conductors have equal resistance and are connected in parallel, the current flowing through resistance R will be equal to half of the total current in the circuit:

$$I_1 = \frac{I}{2} = \frac{\varepsilon}{4r}$$

2) Let's consider connection of the second conductor in series. Then the total external resistance:

$$R_1 = R + R = 2R = 4r$$

According to Ohm's law:

$$I_2 = \frac{\varepsilon}{R_1 + r} = \frac{\varepsilon}{5r}$$

The same current will flow through resistance R as the conductors are connected in series.

3) The ratio of powers will take the following form:

$$\frac{P_1}{P_2} = \frac{I_1^2 R}{I_2^2 R} = \frac{25}{16} = 1.6$$

Remark. Since the smallest number of significant figures is one, the answer 2 is also allowed.

9.9. (7 points) A straight piece of wire of 30 cm length (straight line segment AB) with specific resistance of $3 * 10^{-8}$ Ohm/m is divided by points C, D, E, F so that CD=DE=EF=FB, AC=2CD. Points C, D, E, F, B are connected to point A by segments of wires with different specific resistances so that their resistances are equal to the resistance of the section AC.

[12] Find the total resistance of the circuit between points A and B.

Comment. Give the answer in microOhms. (*Yakovlev A.B.*)

Answer: 0.0015 (possible answer: 0.002).

Solution. 1) The resistance is calculated as

$$R = \rho L,$$

where ρ - resistivity, L - conductor length. Then it is clear that the resistances of sections CD,DE,EF,FB are equal, and the resistance of section AC is twice as large. Let's denote:

$$R_{CD} = R_{DE} = R_{EF} = R_{FB} = R, R_{AC} = 2R$$

According to problem statement, the wire's resistances are equal to:

$$r_{AC} = r_{AD} = r_{AE} = r_{AF} = r_{AB} = R_{AC} = 2R$$

2) To calculate the total resistance of the wire with segments of wires we note that the type of their connection is alternating: R_{AC} and r_{AC} are connected in parallel, they are connected in series with R_{CD} , then all together parallel to r_{AD}, in series with R_{DE} and so on.
Thus we start to calculate the total resistance. For parallel connected R_{AC} and r_{AC} we get that their total resistance is:

$$R_1 = \frac{R_{AC} r_{AC}}{R_{AC} + r_{AC}} = R$$

This section is connected in series with R_{CD}, so the total resistance:

$$R_2 = R_1 + R_{CD} = 2R$$

Then for the parallel connected resistance r_{AD}, the total resistance will be:

$$R_3 = \frac{R_2 r_{AD}}{R_1 + r_{AD}} = R$$

It is clear that further calculations will repeat the calculations above, so at the end we obtain that the total resistance of the system of wires is R
3) The length of the wire section FB is equal to 1/6 of the whole length of the wire. Thus, we obtain:

$$R = \rho l/6 = 0.0015 \mu\Omega$$

Remark. Since the smallest number of significant figures is one, the answer 0.002 $\mu\Omega$ is also allowed.

Solutions to problems for grade R10

10.1. (7 points) At the factory a test is carried out to check the gas flow sensor. It is known that methane flows through a gas pipeline with cross-section of S = 5 cm^2 at pressure of p = 7 atm and temperature of T = 15^o C.

[1] What speed of gas flow should the serviceable sensor display, if a mass of m = 15 kg of methane flows through the cross-section of the pipe in t = 10 min?

Comment. Assume that the molar mass of methane to be M = 16.04 g/mol. (*Cherenkov A.A.*)

Answer: 10.

Solution. 1) The volume of gas flowing through the cross-section of the pipeline:

$$V = vtS$$

2) Let us write down the equation of state of an ideal gas:

$$pV = \frac{m}{M}RT$$

3) Thus, substituting the expression for volume into the Mendeleev-Clapeyron equation, we express the velocity:

$$v = \frac{mRT}{pMtS} = 10.5m/s$$

Since the smallest number of significant figures is equal to one, the answer is $10m/s$

10.2. (5 points) Two vessels of equal volume are filled with oxygen and connected by a tube. The whole system is at temperature of T = 17^o C. At some moment of time one of the vessels is heated to the temperature of $T_1 = 27^o$ C, and the temperature of the second vessel is kept the same.

[2] In how many times will the pressure in the system change?

Comment. Neglect the volume of the tube. (*Cherenkov A.A.*)

Answer: 1.0 (possible answer: 1.02).

Solution. 1) Let us write down the equations of state of gases in heated and non-heated vessels:

$$\begin{cases} pV = \frac{m_1}{M}RT_1 \\ pV = \frac{m_2}{M}RT \end{cases}$$

where m_1,T_1– mass and temperature of gas in the first vessel, m_2,T_2– in the second vessel.
Therefore

$$\begin{cases} m_1 = \frac{pVM}{RT_1} \\ m_2 = \frac{pVM}{RT} \end{cases}$$

2) Let us write down the Mendeleev-Clapeyron equation for the gas in the vessels before heating:

$$2p_0V = \frac{m_1 + m_2}{M}RT$$

where p_0– gas pressure in vessels

3) Let us substitute the expressions for the masses:

$$2p_0 = p(\frac{T}{T_1} + 1)$$

Then we obtain that the pressure ratio is:

$$\frac{p}{p_0} = \frac{2T_1}{T + T_1} = 1.02$$

Since the smallest number of significant figures is two, the answer is 1.0 , i.e. it is impossible to detect the pressure difference with the given accuracy. However, based on the meaning of the question, the answer 1.02 is also allowed.

10.3. (6 points) A straight piece of wire of 30 cm length (straight line segment AB) with specific resistance of $3 * 10^{-8}$ Ohm/m is divided by points C, D, E, F so that CD=DE=EF=FB, AC=2CD. Points C, D, E, F, B are connected to point A by segments of wires with different specific resistances so that their resistances are equal to the resistance of the section AC.

[3] Find the total resistance of the circuit between points A and B.

Comment. Give the answer in microOhms (*Yakovlev A.B.*)
Answer: 0.0015 (possible answer: 0.002).
Solution. 1) The resistance is calculated as

$$R = \rho L,$$

where ρ - resistivity, L - conductor length. Then it is clear that the resistances of sections CD,DE,EF,FB are equal, and the resistance of section AC is twice as large. Let's denote:

$$R_{CD} = R_{DE} = R_{EF} = R_{FB} = R, R_{AC} = 2R$$

According to problem statement, the wire's resistances are equal to:

$$r_{AC} = r_{AD} = r_{AE} = r_{AF} = r_{AB} = R_{AC} = 2R$$

2) To calculate the total resistance of the wire with segments of wires we note that the type of their connection is alternating: R_{AC} and r_{AC} are connected in parallel, they are connected in series with R_{CD} , then all together parallel to r_{AD}, in series with R_{DE} and so on.
Thus we start to calculate the total resistance. For parallel connected R_{AC} and r_{AC} we get that their total resistance is:

$$R_1 = \frac{R_{AC} r_{AC}}{R_{AC} + r_{AC}} = R$$

This section is connected in series with R_{CD}, so the total resistance:

$$R_2 = R_1 + R_{CD} = 2R$$

Then for the parallel connected resistance r_{AD}, the total resistance will be:

$$R_3 = \frac{R_2 r_{AD}}{R_1 + r_{AD}} = R$$

It is clear that further calculations will repeat the calculations above, so at the end we obtain that the total resistance of the system of wires is R
3) The length of the wire section FB is equal to 1/6 of the whole length of the wire. Thus, we obtain:

$$R = \rho l/6 = 0.0015 \mu\Omega$$

Remark. Since the smallest number of significant figures is one, the answer 0.002 $\mu\Omega$ is also allowed.

10.4. (7 points) Pavel conducts an experiment where he drops two identical balls from a height of H = 15 m with no initial velocity, measuring their speeds at the end of the fall and the times taken. One ball hits a platform at a height of h = 10 m, inclined at an angle of $\alpha = 30^o$ to the

horizontal and rebounds elastically, while the other ball falls freely.
[4] How do their speeds differ at the moment of impact with the ground?
[5] How do their fall times differ?
Comment. Assume that the acceleration of free fall is 9.8 m/s^2 (*Cherenkov A.A.*)
Answer: 0.
Answer: 1.3 (possible answer: 1).
Solution. 1) The law of conservation of mechanical energy is satisfied for both balls:

$$mgH = \frac{mv^2}{2}$$

From where we obtain the expression for the velocity:

$$v = \sqrt{2gH}$$

Since the balls were at the same height, their velocities at the end of the path will be the same, so the velocity difference at the end of the path is zero.
2) Let us introduce the y-axis vertically downwards and start counting from the height H.
Law of motion of a ball with no obstacle on its path:

$$y_1(t) = \frac{gt^2}{2}$$

When ball falls on the ground: $y_1(T_1) = H$, where T_1 is the total time of motion of the ball. Thus we obtain that

$$T_1 = \sqrt{\frac{2H}{g}} \approx 1.7s$$

Equation of motion of the second ball before hitting the obstacle:

$$y_2(t) = \frac{gt^2}{2}$$

In this case, the velocity changes according to the following law:

$$v_2(t) = gt$$

At the moment t_2 of impact with the obstacle the ball will be at height h, i.e. $y_2(t_2) = H - h$, from whence we obtain that

$$t_2 = \sqrt{\frac{2(H-h)}{g}} \approx 1s$$

and the velocity at this moment will be equal to:

$$v_2(t_2) = g\sqrt{\frac{2(H-h)}{g}} = \sqrt{2g(H-h)} = v_0$$

In case of elastic impact with an obstacle, the modulus of velocity does not change, only its direction changes. From geometrical considerations we find that the ball will reflect from the obstacle at an angle of

$$\beta = 90^o - 2\alpha = 90^o - 2 * 30^o = 30^o = \alpha$$

.

Then the equation of motion of the ball has the following form:

$$y_2(t) = (H - h) - v_0 \sin\alpha t + \frac{gt^2}{2}$$

At the moment τ_2 when the ball falls to the ground $y_2(\tau_2) = H$. Then we obtain the quadratic equation for determining τ_2:

$$H = (H - h) - v_0 \sin\alpha\tau_2 + \frac{g\tau_2^2}{2}$$

from whence we find that

$$\tau_2 = \frac{1}{g}(v_0 \sin\alpha \pm \sqrt{(v_0 \sin\alpha)^2 + 2gh})$$

Taking into account that $\tau_2 > 0$, we get:

$$\tau_2 = \frac{1}{g}(v_0 \sin\alpha + \sqrt{(v_0 \sin\alpha)^2 + 2gh}) \approx 2s$$

Thus, the total time of motion of the ball:

$$T_2 = t_2 + \tau_2 \approx 3s$$

And the time difference:

$$\Delta t = T_2 - T_1 \approx 1.3s$$

10.5. (6 points) Petya has a dream in which he is on a desert island. To get food, he goes hunting with a bow. Walking into the jungle, Petya notices a tree at a distance of L = 10 m from him, with a monkey sitting on a branch at a height of H = 5 m. Drawing the bowstring, Petya shoots an arrow. The monkey, who was resting on the tree, started to fall down from fright at the same moment.

[6] What should be the minimum speed of the arrow so that Petya hits the monkey?

Comment. Assume that the acceleration of free fall is 9.8 m/s^2. (*Cherenkov A.A.*)
Answer: 11 (possible answer: 10).
Solution. 1) Let's introduce a stationary coordinate system associated with the Earth at the point where Petya is standing. Let us also introduce a moving coordinate system rigidly connected with the falling monkey.
Let's consider the laws of change of velocities of the arrow and the monkey in the stationary coordinate system:

$$\begin{cases} \vec{v}(t) = \vec{v}_0 + \vec{g}t \\ \vec{u}(t) = \vec{g}t \end{cases}$$

where $\vec{v}, \vec{u}$ – velocities of the monkey and the arrow, respectively. Then in the moving coordinate system, by the velocities addition law, the velocity of the arrow is:

$$\vec{v}_r = \vec{v} - \vec{u} = \vec{v}_0 = (\overrightarrow{const})$$

Thus, Petya will hit the monkey if he aims directly at it at any sufficiently high speed. According to the problem, Petya is aiming directly at the monkey. Then it is clear that the minimum speed at which Petya will hit the monkey will be when the arrow reaches the monkey at the same moment that it falls to the Earth.
2) Let's find by Pythagoras' theorem the initial distance between Petya and the monkey:
$l = \sqrt{H^2 + L^2}$.
Let α be the angle at which the arrow is shot. Then $\sin\alpha = \frac{H}{l}, \cos\alpha = \frac{L}{l}$ Let us write down the equations of motion of the arrow in the previously introduced stationary Cartesian coordinate system:

$$\begin{cases} x(t) = v_0 \cos \alpha t \\ y(t) = v_0 \sin \alpha t - \frac{gt^2}{2} \end{cases}$$

The arrow will travel a distance L horizontally when it hits the monkey at time T, i.e. $x(T) = L$, from where we get:

$$L = v_0 \cos \alpha T$$

then

$$T = \frac{L}{v_0 \cos \alpha}$$

At the moment of time T the arrow will fall to the Earth, i.e. $y(T) = 0$. Therefore:

$$0 = v_0 \sin \alpha T - \frac{gT^2}{2}$$

Divide this equation by $T \neq 0$ and solve it with respect to T:

$$T = \frac{2v_0 \sin \alpha}{g}$$

Equate the two obtained expressions for T:

$$\frac{2v_0 \sin \alpha}{g} = \frac{L}{v_0 \cos \alpha}$$

Whence we get:

$$v_0 = \sqrt{\frac{Lg}{2 \cos \alpha \sin \alpha}} = \sqrt{\frac{Lg(H^2 + L^2)}{2HL}} = \sqrt{\frac{g(H^2 + L^2)}{2H}} \approx 11.1 m/s$$

According to the rule of determining significant figures when extracting a root, the correct answer is $11 m/s$

Remark. Since the smallest number of significant digits is equal to one, the answer 10 m/s is also allowed.

10.6. (7 points) Sasha invented a method which calculates the velocities of bodies. For this purpose he took an inclined plane and put a notch on it, at a distance of L = 10 cm from the base. Then he rolled the ball from bottom to top and measured the times t_1 = 2 s and t_2 = 5 s from the beginning of movement, when the ball passed the notch. Thus he was able to find out what velocity the ball had at the beginning of the motion.

[7] What was this velocity equal to?

[8] At what minimum angle in degrees of inclined plane Sasha's method fails if the position of the notch and the initial velocity are the same?

Comment. Assume that the acceleration of free fall is 9.8 m/s^2. (*Cherenkov A.A.*)

Answer: 0.07.

Answer: 0.14.

Solution. 1) Let us introduce a frame of reference along an inclined plane with the x-axis pointing upwards with the origin at the lower point of the plane . Then the law of motion of the ball along the plane:

$$x(t) = v_0 t - \frac{at^2}{2}$$

Hence we obtain that

$$t^2 - \frac{2v_0}{a} t + \frac{2x}{a} = 0$$

Since t_1 and t_2 - are the roots of this equation at $x = L$, then according to Vieta's theorem:

$$\begin{cases} t_1 + t_2 = \frac{2v_0}{a} \\ t_1 \cdot t_2 = \frac{2L}{a} \end{cases}$$

Solving the system, we find that

$$v_0 = \frac{L}{t_1} + \frac{L}{t_2} = 7cm/s$$

2) Sasha's method will not work if the ball does not reach the notch on the inclined plane. Therefore, the minimum required angle of the inclined plane is determined from the condition that the ball reaches the notch and stops, continuing its movement down the inclined plane.
Let us write down the law of velocity change of the ball:

$$v(t) = v_0 - g \sin \alpha t$$

Let the stop of the ball occurs at time T, then:

$$0 = v_0 - g \sin \alpha T$$

then

$$T = \frac{v_0}{g \sin \alpha}$$

Notice that x(T)=L, then after substituting T into the law of motion we obtain:

$$x(T) = L = \frac{v_0^2}{2g \sin \alpha}$$

Whence we finally get that

$$\sin \alpha = \frac{v_0^2}{2Lg} = \frac{1}{400}$$

then

$$\alpha \approx 0.14^o$$

Since the smallest number of significant figures in item 1 is one and the answer must be given in the SI system, the answer in item 1 is 0.07 m/s. In item 2, the smallest number of significant digits is two.

10.7. (5 points) Petya was invited to go karting for his friend's birthday. Karting cars reach a maximum speed of 50 km/h. Petya drove the first part of the track, accelerating to 25 km/h. The rest of the track Petya covered after accelerating to the maximum speed.

[9] In how many times the work of the engine during acceleration on the second part of the track is greater than on the first part?

Comment. Consider the acceleration time and the drag force to be the same in both cases.

(*Cherenkov A.A.*)

Answer: 3.

Solution. Solution: 1) Write down Newton's second law:

$$F_{tr} - F_{res} = ma$$

therefore

$$F_{tr} = F_{res} + ma = const$$

2) Calculate the distance travelled on the first and second half of the track:

$$s_1 = \frac{v^2}{2a}, \quad s_2 = \frac{(2v)^2 - v^2}{2a} = 3s_1,$$

Where $v = 25km/h$
3) Thus, the ratio of work done by the engine:

$$\frac{A_2}{A_1} = \frac{F_{tr}s_2}{F_{tr}s_1} = 3$$

10.8. (6 points) At the construction site of a kindergarten a rope of length L = 5 m was hanging on a pin. One of the workers, passing by, accidentally touched it, and the rope began to move.

[10] What speed will the rope have when it slips completely off the pin, if rope's ends had been at the same level before it started to move?

Comment. Assume that the acceleration of free fall is 9.8 m/s^2. (*Cherenkov A.A.*)
Answer: 4.9 (possible answer: 5).
Solution. 1)At the initial moment the centre of gravity of the rope was at a distance$l_1 = \frac{L}{4}$ from the pin, and when the rope left the pin - $l_2 = \frac{L}{2}$
2) By the law of conservation of energy:

$$-mgl_1 = \frac{mv^2}{2} - mgl_2$$

From where we get:

$$v = \sqrt{\frac{gL}{2}} = 4.9m/s$$

According to the rule of significant figures when extracting the root, the correct answer is 4.9 m/s.

10.9. (7 points) The sled slides down a mountain with an inclination angle $\beta = 30^o$, and moves sequentially along a horizontal section of length $s_1 = 7$ m, through a hill of h = 3 m height and inclination angle of $\alpha = 60^o$ and again along a horizontal section. The coefficient of friction on horizontal sections is $\mu_1 = 0.1$, on inclined sections - $\mu_2 = 0.3$.

[11] Determine the height from which the sled should start so that it would travel along the second horizontal section at least $s_2 = 15$ metres.

(*Cherenkov A.A.*)

Answer: 9.
Solution. 1) Calculate the lengths of the inclined sections along which the sled moves. When descending the first hill:

$$l = \frac{H}{\sin\beta}$$

When descending the second hill:

$$l_1 = \frac{h}{\sin\alpha}$$

2)Let A_1,A_2,A_3,A_4 be the magnitudes of the work done by friction forces while moving along the first inclined section, the first horizontal section, the second inclined section, and the second horizontal section, respectively. Clearly, when moving on the second hill, the friction force does the same work A_3 on both slopes. Let's calculate these works. Consider the horizontal sections. According to Newton's second law along the vertical axis:

$$mg = N$$

Therefore:

$$F = \mu_1 N = \mu_1 mg$$

then

$$A_2 = \mu_1 mgs_1, A_4 = \mu_1 mgs,$$

where s is the distance traveled by the sled on the second horizontal section. Now, consider the inclined sections. For the sled moving down the first hill, according to Newton's second law along the axis perpendicular to the slope surface:

$$N = mg\cos\beta$$

Therefore:

$$F = \mu_2 N = \mu_2 mg\cos\beta$$

then

$$A_1 = \mu_2 mgl\cos\beta$$

Similarly, we find $A_3 = \mu_2 mgl_1 \cos\alpha$

3) First, let us determine the distance traveled by the sled before coming to a complete stop on the second hill, assuming zero initial velocity. From the law of energy conservation:

$$mgh = A_3 + A_4$$

$$mgh = \mu_2 mgl_1\cos\alpha + \mu_1 mgs$$

Therefore

$$s = \frac{h - \mu_2 l_1 \cos\alpha}{\mu_1} = \frac{1 - \mu_2 \operatorname{ctg}\alpha}{\mu_1} h$$

Thus, the sled will travel the given distance on the second horizontal section regardless of its initial speed. Therefore, the problem reduces to finding the height from which the sled must descend to reach the top of the second hill.

4) Using the law of energy conservation for the descent from the first hill:

$$mgH = A_1 + A_2 + A_3 + mgh$$

$$mgH = \mu_2 mgl\cos\beta + \mu_1 mgs_1 + \mu_2 mgl_1\cos\alpha + mgh$$

From where we obtain:

$$H = \mu_2 l\cos\beta + \mu_1 s_1 + \mu_2 l_1 \cos\alpha + h = \mu_2 H \operatorname{ctg}\beta + \mu_1 s_1 + \mu_2 h \operatorname{ctg}\alpha + h$$

Final expression for the height:

$$H = \frac{\mu_1 s_1 + (\mu_2 \operatorname{ctg}\alpha + 1)h}{1 - \mu_2 \operatorname{ctg}\beta} = 8.8m$$

Since the smallest number of significant figures is one, the correct answer is 9 m .

Solutions to problems for grade R11

11.1. (6 points) . In the school physics circle Petya was charged to construct temperature dependences in the temperature range from $T_1 = 20^o$ C to $T_2 = 50^o$ C for a system, consisting of a coal rod of length $l_1 = 3$ cm and radius of r = 1 mm and a metal rod of the same radius and length $l_2 = 20, 60, 80, 90$ cm.

[1] Petya found out that in one of these cases there is no temperature dependence. What is the length of the metal rod in centimeters in this case?

Comment. Temperature coefficients and resistivity at 0^o C for coal and metal are equal to $\alpha_1 = -0{,}8 * 10^{-3}$ K^{-1}, $\varrho_1 = 4 * 10^{-5}$ Ohm*m, $\alpha_2 = 6 * 10^{-3}$ K^{-1}, $\varrho_2 = 2 * 10^{-7}$ Ohm*m. (*Yakovlev A.B.*)

Answer: 80.

Solution. 1) The carbon and metal rods are connected in series, so their total resistance at temperature T will be equal to:

$$R = R_1(1 + \alpha_1 T) + R_2(1 + \alpha_2 T) = (R_1\alpha_1 + R_2\alpha_2)T + R_1 + R_2$$

where $R_1 = \frac{\rho_1 l_1}{S_1}$ и $R_2 = \frac{\rho_2 l_2}{S_2}$ - resistances of the carbon and metal rods at T = 0, respectively
2) The resistance of the rods will not depend on the temperature if

$$(R_1\alpha_1 + R_2\alpha_2) = 0$$

From where we get

$$\frac{\rho_1 l_1}{S_1}\alpha_1 = -\frac{\rho_2 l_2}{S_2}\alpha_2$$

So the length of the metal rod:

$$l_2 = -\frac{\rho_1 S_2 \alpha_1}{\rho_2 S_1 \alpha_2} l_1 = -\frac{\rho_1 \alpha_1}{\rho_2 \alpha_2} l_1 = 80cm$$

11.2. (9 points) A cannon fires a projectile at an angle of $\alpha = 30^o$ to the horizon with an initial velocity of v = 20 m/s. At the peak of its trajectory, projectile explodes into fragments flying in all directions, each having the same relative speed $v_0 = 5$ m/s with respect to the projectile.

[2] Find the volume bounded by the fragments $t_0 = 1$ s after the explosion.

[3] What will be the maximum velocity,

[4] minimum velocity of the fragments relative to the Earth after t_0?

Comment. Neglect the air resistance. Assume that the acceleration of free fall is 9.8 m/s^2.
(*Cherenkov A.A.*)

Answer: 500.
Answer: 25.
Answer: 15.

Solution. 1) From the law of motion of the center of mass, it is clear that after the explosion, the entire system of fragments will continue to move in such a way that its center of mass follows the same trajectory as the projectile would have if it had not exploded.
Thus, we introduce a moving coordinate system oxy rigidly attached to the center of mass of the fragment system and examine the motion of an arbitrary fragment, whose initial relative velocity was directed at an angle β to the horizontal. Then the equations of motion for this fragment are:

$$\begin{cases} x(t) = v_0 \cos\beta t \\ y(t) = v_0 \sin\beta t \end{cases}$$

From these equations we obtain:

$$\begin{cases} \frac{x(t)}{v_0 t} = \cos\beta \\ \frac{y(t)}{v_0 t} = \sin\beta \end{cases}$$

Taking into account the basic trigonometric identity, we find:

$$\frac{1}{v_0^2 t^2}(x^2 + y^2) = 1$$

Therefore

$$x^2 + y^2 = v_0^2 t^2$$

This means that throughout the motion, the fragments are located on the surface of a sphere with radius $R(t) = v_0 t$, centered at the origin of the moving coordinate system oxy. At time t_0 the fragments bound a volume equal to the volume of a sphere with radius $R(t_0) = v_0 t_0$:

$$V = \frac{4}{3}\pi(v_0 t_0)^3 = \frac{4}{3}\pi(v_0 t_0)^3 \approx 523.6 m^3$$

2) By the law of velocity addition,

$$\vec{v}_a = \vec{v}_r + \vec{v}_e,$$

Where v_a,v_r are absolute and relative velocities respectively, v_e is the velocity of the moving coordinate system (transport velocity). The fragment with the highest absolute velocity is the one whose relative velocity is aligned with the transport velocity. Conversely, the fragment with the lowest velocity has a relative velocity opposite to the transport velocity. Therefore:

$$v_{max} = v_r + v_e, v_{min} = |v_r - v_e|$$

It is important to note that the relative velocities of all fragments are equal to v_0 at all times. Next, we calculate the transport velocity at $t_0 = 1c$ after the explosion. It equals the velocity of the projectile at this time as if it had not exploded. Introducing a stationary coordinate system at the point of launch, the velocity components of the projectile are:

$$\begin{cases} v_x(t) = v\cos\alpha \\ v_y(t) = v\sin\alpha - gt \end{cases}$$

At the highest point of the trajectory, the vertical velocity component is zero:

$$0 = v_y(T) = v\sin\alpha - gT,$$

from where

$$T = \frac{v\sin\alpha}{g}$$

Then the transport velocity is:

$$v_e(t_0+T) = \sqrt{v_x^2(t_0+T) + v_y^2(t_0+T)} = \sqrt{(v\cos\alpha)^2 + (v\sin\alpha - g(t_0 + \frac{v\sin\alpha}{g}))^2} = \sqrt{(v\cos\alpha)^2 + (gt_0)^2}$$

Finally, the expressions for the maximum and minimum velocities of the fragments are:

$$v_{max} = v_r + v_e = v_0 + \sqrt{(v\cos\alpha)^2 + (gt_0)^2} \approx 24.9 m/s$$
$$v_{min} = |v_0 - \sqrt{(v cos\alpha)^2 + (gt_0)^2}| = 14.9 m/s$$

Given that the least number of significant figures is one, the correct answer for part 1 is 500 m . For part 2, based on the rules for determining significant figures when taking square roots, the correct answers are $25m/s; 15m/s$.

11.3. (5 points) Danya has a toy railway on which a train can travel at a speed of 25 km/h. He has attached a voltmeter to the rails.

[5] What will be the reading of the voltmeter when the train approaches it if the distance between the rails is d = 10 cm?

Comment. Consider that the normal component of the Earth's magnetic induction $B_n = 4 * 10^{-5}$ Tesla. (*Cherenkov A.A.*)

Answer: 0.00003.

Solution. 1) According to Faraday's law of electromagnetic induction, the EMF induced in a circuit is related to the change in magnetic flux:

$$\varepsilon = \frac{\Delta\Phi}{\Delta t} = \frac{B_n \Delta S}{\Delta t}$$

2) Let us express the change of the contour area per unit of time through the train speed:

$$\Delta S = v\Delta t d$$

3) Finally we get:

$$\varepsilon = \frac{B_n v \Delta t d}{\Delta t} = B_n v d = 27.8\mu V$$

Since the smallest number of significant figures is equal to one and the answer must be given in the SI system, the answer is 0.00003 V.

11.4. (9 points) Petya is going to take part in a rocket-building championship. The boy has built a test model of a rocket with mass of M = 2 kg and decided to test it. The rocket is launched from the ground with an initial velocity of $v_0 = 25$ m/s at an angle of $\alpha = 30^o$ to the horizon. Three times at equal intervals of time $\Delta t = 0.3$ s a mass of $\Delta m = 0.5$ kg is ejected from the rocket with velocity of u = 5 m/s relative to the rocket.

[6] What speed will the rocket have when it approaches the Earth?

Comment. Neglect the air resistance. Assume that the acceleration of free fall is 9.8 m/s^2. (*Cherenkov A.A.*)

Answer: 33 (допустим ответ 30).

Solution. 1) Let us introduce a stationary rectangular coordinate system oxyoxyoxy at the rocket's launch site. Then, the laws governing the changes in coordinates and velocities are as follows:

$$\begin{cases} y(t) = v_0 \sin\alpha t - \frac{gt^2}{2} \\ v_x(t) = v_0 \cos\alpha \\ v_y(t) = v_0 \sin\alpha - gt \end{cases}$$

At time Δt we will have:

$$\begin{cases} y(\Delta t) = v_0 \sin\alpha \Delta t - \frac{g\Delta t^2}{2} = y_1 \approx 3.31m \\ v_x(\Delta t) = v_0 \cos\alpha = v_{1x} \approx 21.65m/s \\ v_y(\Delta t) = v_0 \sin\alpha - g\Delta t = v_{1y} \approx 9.56m/s \end{cases}$$

At this time, a mass Δm will be ejected from the rocket. Let us calculate the new velocity components of the rocket v_{01x},v_{01y} after the ejection. We apply the law of conservation of momentum, projected onto the axes:

$$\begin{cases} Mv_{1x} = (M - \Delta m)v_{01x} + \Delta m(v_{1x} - u_x) \\ Mv_{1y} = (M - \Delta m)v_{01y} + \Delta m(v_{1y} - u_y) \end{cases}$$

Where $u_x = u\frac{v_{1x}}{\sqrt{v_{1x}^2+v_{1y}^2}}$,$u_y = u\frac{v_{1y}}{\sqrt{v_{1x}^2+v_{1y}^2}}$ - projections of the velocity of the ejected mass
Then solving the system, we obtain:

$$\begin{cases} v_{01x} = v_{1x} + \frac{\Delta m}{M-\Delta m}u\frac{v_{1x}}{\sqrt{v_{1x}^2+v_{1y}^2}} \approx 23.17m/s \\ v_{01y} = v_{1y} + \frac{\Delta m}{M-\Delta m}u\frac{v_{1y}}{\sqrt{v_{1x}^2+v_{1y}^2}} \approx 10.23m/s \end{cases}$$

2) Let us now write the new equations of motion for the rocket, resetting the time to zero. We obtain:

$$\begin{cases} y(t) = y_1 + v_{01y}t - \frac{gt^2}{2} \\ v_x(t) = v_{01x} \\ v_y(t) = v_{01y} - gt \end{cases}$$

At time Δt we will have:

$$\begin{cases} y(\Delta t) = y_1 + v_{01y}\Delta t - \frac{g\Delta t^2}{2} = y_2 \approx 5.94m \\ v_x(\Delta t) = v_{01x} = v_{2x} \approx 23.17m/s \\ v_y(\Delta t) = v_{01y} - g\Delta t = v_{2y} \approx 7.29m/s \end{cases}$$

Following a similar reasoning as in part 1, using the law of conservation of momentum, we can find the new velocity components of the rocket v_{02x},v_{02y} after another mass ejection:

$$\begin{cases} (M - \Delta m)v_{2x} = (M - 2\Delta m)v_{02x} + \Delta m(v_{2x} - u_x) \\ (M - \Delta m)v_{2y} = (M - 2\Delta m)v_{02y} + \Delta m(v_{2y} - u_y) \end{cases}$$

Where $u_x = u\frac{v_{2x}}{\sqrt{v_{2}x^2+v_{2}y^2}}$,$u_y = u\frac{v_{2y}}{\sqrt{v_{2x}^2+v_{2y}^2}}$ - projections of the velocity of the ejected mass
Then solving the system we get:

$$\begin{cases} v_{02x} = v_{2x} + \frac{\Delta m}{M-2\Delta m}u\frac{v_{2x}}{\sqrt{v_{2x}^2+v_{2y}^2}} \approx 25.56m/s \\ v_{02y} = v_{2y} + \frac{\Delta m}{M-2\Delta m}u\frac{v_{2y}}{\sqrt{v_{2x}^2+v_{2y}^2}} \approx 8.04m/s \end{cases}$$

3) Now, we will write the new equations of motion for the rocket after the second mass ejection, resetting the time to zero. The equations are:

$$\begin{cases} y(t) = y_2 + v_{02y}t - \frac{gt^2}{2} \\ v_x(t) = v_{02x} \\ v_y(t) = v_{02y} - gt \end{cases}$$

At time Δt we will have:

$$\begin{cases} y(\Delta t) = y_2 + v_{02y}\Delta t - \frac{g\Delta t^2}{2} = y_3 \approx 7.91m \\ v_x(\Delta t) = v_{02x} = v_{3x} \approx 25.56m/s \\ v_y(\Delta t) = v_{02y} - g\Delta t = v_{3y} \approx 5.10m/s \end{cases}$$

Following a similar reasoning as in part 1, using the law of conservation of momentum, we can find the new velocity components of the rocket v_{03x},v_{03y} after another mass ejection:

$$\begin{cases} (M - 2\Delta m)v_{1x} = (M - 3\Delta m)v_{01x} + \Delta m(v_{1x} - u_x) \\ (M - 2\Delta m)v_{1y} = (M - 3\Delta m)v_{01y} + \Delta m(v_{1y} - u_y) \end{cases}$$

Where $u_x = u\frac{v_{3x}}{\sqrt{v_{3x}^2+v_{3y}^2}}$,$u_y = u\frac{v_{3y}}{\sqrt{v_{3x}^2+v_{3y}^2}}$ - projections of the velocity of the ejected mass
Then solving the system we get:

$$\begin{cases} v_{03x} = v_{3x} + \frac{\Delta m}{M-3\Delta m}u\frac{v_{3x}}{\sqrt{v_{3x}^2+v_{3y}^2}} \approx 30.46m/s \\ v_{03y} = v_{3y} + \frac{\Delta m}{M-3\Delta m}u\frac{v_{3y}}{\sqrt{v_{3x}^2+v_{3y}^2}} \approx 6.09m/s \end{cases}$$

The total velocity is calculated as:

$$v_3 = \sqrt{v_{03x}^2 + v_{03y}^2}$$

4) Finally, to calculate the velocity of the rocket as it approaches the Earth, we write down the energy conservation law:

$$\frac{\tilde{M}v^2}{2} = \frac{\tilde{M}v_3^2}{2} + \tilde{M}gy_3,$$

where $\tilde{M}$ - current mass of the rocket
Therefore:

$$v = \sqrt{v_3^2 + 2gy_3} = \sqrt{v_{03x}^2 + v_{03y}^2 + 2gy_3} \approx 33.46m/s$$

According to the rule of determining significant figures when extracting the root, the correct answer is 33 m/s . Since the smallest number of significant figures is equal to one, the acceptable answer is 30 m/s.

11.5. (8 points) There are two ramps in a skatepark - one is fixed to the ground and stationary, and the other is on a movable support. Both ramps are of the same height h = 2 m. A local skateboarder first rolled down the first ramp and then down the second ramp.

[7] In how many times will the speed of the skateboarder change in the second case compared to the first case, if the mass of the skateboarder is twice less than the mass of the ramp?

Comment. Neglect friction. Assume that the acceleration of free fall is 9.8 m/s^2. (*Cherenkov A.A.*)
Answer: 1.2.
Solution. 1) In the first case the ramp remains stationary, then we write the energy conservation law:

$$\frac{mv^2}{2} = mgh$$

From where we find the speed of the skateboarder at the end of the descent:

$$v = \sqrt{2gh}$$

2) In the second case, when the skateboarder descends, the ramp goes into motion. Let us write the momentum conservation law on the X-axis:

$$mv_1 = MV$$

Whence the velocity that the ramp acquires:

$$V = \frac{mv_1}{M}$$

According to the energy conservation law:

$$mgh = \frac{mv_1^2}{2} + \frac{MV^2}{2}$$

$$2gh = v_1^2 + \frac{m}{M}v_1^2$$

Whence the speed of the skateboarder at the end of the descent:

$$v_1 = \sqrt{\frac{2gh}{1+\frac{m}{M}}}$$

3) Thus, we find the ratio of velocities:
$\frac{v_1}{v} = \sqrt{1+\frac{m}{M}} = 1.22$
According to the rule of significant figures when taking the root, the correct answer is 1.2.

11.6. (7 points) The equilibrium of solids in liquids is studied in a school laboratory. The teacher took an empty cylindrical glass and carefully immersed it upwards with the bottom and released it. The glass occurred to be in the state of equilibrium.

[8] What is the depth of immersion of the glass?

Comment. The glass has height of H = 15 cm, diameter of D = 3 cm and mass of m = 0.1 kg. Assume that atmospheric pressure is $p_0 = 10^5$ Pa and acceleration of free fall is g = 9.8 m/s^2. The density of water is $\rho = 1000\ kg/m^3$. (*Cherenkov A.A.*)
Answer: 0.6.
Solution. 1) Water pressure at depth h:

$$p_1 = p_0 + \rho gh,$$

where ρ – water density, p_0 - atmospheric pressure
2) The Boyle-Marriott law is true for the air in the glass:

$$p_0V_0 = p_1V_1$$

where V_0,V_1- volumes of air in the glass before and after immersion respectively
3) Consider that the glass is in equilibrium, supported by the Archimedean force of the air in it:

$$\rho gV_1 = mg$$

4) Solving the system of equations 1-3, we obtain the depth:

$$h = \frac{p_0}{mg}\left(V_0 - \frac{m}{\rho}\right) = \frac{p_0}{mg}\left(\frac{\pi D^2}{4}H - \frac{m}{\rho}\right) = 0.6m$$

11.7. (7 points) Petya read in the textbook a method of determining the charge of a drop. It is needed to take a flat capacitor and measure the times of drop falling from one plate of capacitor to the other at different potential differences. The first time Petya applied a potential difference of 100 V and the second time - 200 V. The times he measured were $t_1 = 2$ s $t_2 = 3$ s.

[9] What is the modulus of charge of a drop if its mass m = 50 mg?

Comment. Assume that the acceleration of free fall is 9.8 m/s^2. (*Cherenkov A.A.*)
Answer: 0.00002.
Solution. 0) Note that as the potential difference between the capacitor plates increases, the fall time of the drop increases. This means that the Coulomb force acting on the drop is directed vertically upwards.
1) Both gravity and Coulomb's force act on the drop. Let us write Newton's second law in projection on the vertical axis:

$$mg - F_k = ma$$

Therefore:

$$a = g - \frac{F_k}{m} = g - \frac{Eq}{m} = g - \frac{Uq}{dm},$$

where E,U,d – electric field, voltage and distance between the capacitor plates respectively, q is the charge of the drop.
2) Then the equation of motion of a drop in projection on the vertical axis has the form:

$$y(t) = \frac{at^2}{2}$$

At the moment T, when the drop finished falling:

$$y(T) = d = \frac{aT^2}{2} = \frac{T^2}{2}(g - \frac{Uq}{dm})$$

3) Writing the last equation for the first and the second case of drop falling, we obtain the system of equations:

$$\begin{cases} d = \frac{t_1^2}{2}(g - \frac{U_1 q}{dm}) \\ d = \frac{t_2^2}{2}(g - \frac{U_2 q}{dm}) \end{cases}$$

To solve the system, first multiply each equation by $2dm$ and equate the right-hand sides:

$$t_1^2(gdm - U_1 q) = t_2^2(gdm - U_2 q)$$

From where we get the expression for d:

$$d = \frac{q(U_2 t_2^2 - U_1 t_1^2)}{gm(t_2^2 - t_1^2)}$$

Let us now substitute this expression into the first equation:

$$\frac{q(U_2 t_2^2 - U_1 t_1^2)}{gm(t_2^2 - t_1^2)} = \frac{t_1^2 g}{2}\left(1 - \frac{U_1(t_2^2 - t_1^2)}{U_2 t_2^2 - U_1 t_1^2}\right)$$

And finally we get the expression for q:

$$q = g^2 t_1^2 t_2^2 m \frac{(U_2 - U_1)(t_2^2 - t_1^2)}{(2 * (U_2 t_2^2 - U_1 t_1^2)^2} \approx 0.000022C$$

Since the smallest number of significant figures is equal to one and the answer must be given in the SI system, the answer is 0.00002 C.

11.8. (5 points) A copper ball of radius 1 mm is suspended on a string over a grounded unbounded flat metallic surface. The distance between the ball and the surface is l = 5 cm. The ball is given some charge.

[10] In how many times will the interaction force between the plate and the ball change, if the distance between them is increased by $l_0 = 2$ cm.

(*Cherenkov A.A.*)

Answer: 0.5.

Solution. 1) The action of a conducting plane with its induced charges can be replaced by the action of a point charge, which is a mirror image of the given charge in the conducting plane. Then the force acting on the charge located at a distance l from the plane:

$$F = k\frac{q^2}{(2l)^2}$$

2) Thus, the ratio of forces acting on the charge:

$$\frac{F_2}{F_1} = \frac{l_1^2}{l_2^2} = \frac{l^2}{(l+l_0)^2} = 0.5$$

11.9. (7 points) Pupils of one school were invited on a tour to a physics laboratory. At one of the installations, the children were shown an experiment demonstrating the impact of an electric field on an electron moving in it. An electron with an energy of 1000 eV flew into a flat capacitor at an angle of $\alpha_1 = 30^o$ to its plates, and flies out at an angle of $\alpha_2 = 60^o$.

[11] What was the voltage on the capacitor, if its length is l = 10 cm and the distance between the plates is d = 1 cm?

Comment. Neglect the effect of gravity. (*Cherenkov A.A.*)

Answer: 200.

Solution. 1) Calculate the initial velocity of the electron:

$$W = \frac{mv_0^2}{2}$$

then

$$v_0^2 = \frac{2W}{m}$$

2) Let us introduce a rectangular coordinate system. The x-axis is directed along the plates of the capacitor, and the y-axis is perpendicular to the surface of the plates of the capacitor. Let us write down Newton's second law in projection on the y-axis:

$$F = ma$$

then

$$a = \frac{F}{m} = \frac{Eq}{m} = \frac{Uq}{dm}$$

3) The equation of motion of the electron along the capacitor:

$$x(t) = v_x t$$

The electron velocity projections are:

$$\begin{cases} v_x(t) = v_0 \cos\alpha_1 \\ v_y(t) = v_0 \sin\alpha_1 + at \end{cases}$$

Then the time needed for the electron to pass the capacitor:

$$T = \frac{x(T)}{v_x(T)} = \frac{l}{v_0 \cos\alpha_1}$$

4) By the condition, the electron flies out at an angle of α_2, then:

$$\operatorname{tg}\alpha_2 = \frac{v_y(T)}{v_x(T)} = \frac{v_0 \sin\alpha_1 + aT}{v_0 \cos\alpha_1}$$

Whence we finally find U, substituting expressions for time T and acceleration a:

$$U = (\operatorname{tg}\alpha_2 - \operatorname{tg}\alpha_1)\frac{2W \cos\alpha_1^2 d}{ql}$$

The calculation is carried out by taking 1eV as the unit of energy, then the charge of the electron is equal to one. We get:

$$U = 173V$$

Since the smallest number of significant digits is one, the answer is 200 V.