International Mathematical Olympiad
"Formula of Unity" / "The Third Millennium"
Year 2024/2025. Qualifying round

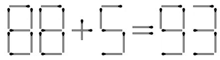
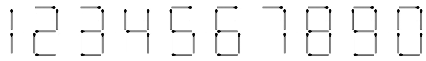
Problems for grade R5

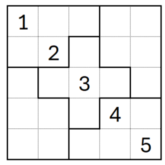
Please hand in your paper in electronic form (e. g. as a doc-file with text or as a scan), some details are at the page formulo.org/en/olymp/2024-math-en/. Your paper should be sent until **23:59:59 UTC, 10 November 2024**.

Please solve the problems by yourself. Remember that the majority of the problems require not only an answer but also its full proof. The paper should not contain your personal data, so **please do not sign your paper**.

---

1. Is it possible to place distinct one-digit numbers in a 2×4 rectangle so that the sum of any two numbers which are adjacent by side is a prime number?
   Remark. We remind that a prime number is an integer greater than 1 that is divisible only by 1 and itself. (*M. Karlukova*)

2. First grader Paul laid out the equation $88 + 5 = 93$ using matchsticks (see on the right). The teacher gave him an 'excellent' grade.
   (a) Help first grader Andrew also get an 'excellent' grade by moving exactly one matchstick in Paul's example so that the equation remains valid.
   
   (b) Come up with your own valid example using 5 different digits laid out with matchsticks, so that one matchstick can be moved from one digit to another to obtain a valid equation again (in other words, the arithmetic operation signs cannot be changed, and the sign «=» cannot be crossed out). Samples of all digits are shown on the right. (*P. Mulenko*)
   

3. How many solutions does the figure sudoku shown on the right have? Don't forget not only to find all the solutions but to explain that you have found all of them.
   

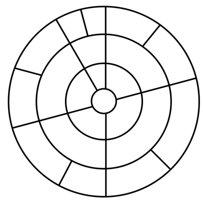
   Remark. According to the rules of a 5×5 figure sudoku, all digits from 1 to 5 must appear in each row, column, and each highlighted block. (*M. Karlukova*)

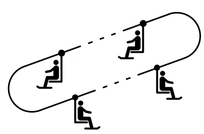
4. Connor and Mary take turns placing the signs + and − in the squares of a chessboard (Connor starts) according to the following rules.
   - Each turn, the player chooses any free square and places one sign of their choice in it.
   - If, after a player's move, there is an equal number of pluses and minuses in squares of the chosen color, that player automatically loses.
   - After the board is filled, it is determined (for black and white squares separately) which signs are in greater number. If one color has more pluses and the other has more minuses, then Mary wins; otherwise, Connor wins.

   Who can ensure their victory, and how should they act to win? (*P. Mulenko*)

5. Several children came to participate in the final stage of an Olympiad.
   - Among them, there are two girls, each of whom knows exactly four boys.
   - There are five girls, each of whom knows exactly two boys.
   - Each of the remaining girls (if any) knows exactly three boys.
   - Each boy knows four girls.

   What is the minimum number of girls who could have come to the Olympiad? Don't forget to prove that this is indeed the minimum. (*L. Koreshkova*)

6. The company's office is located on a separate floor of a business center (see the figure). Each room contains a department managed by a manager. The company director decided to promote some department heads to chief managers. However, it is not allowed for two neighboring departments to be managed by chief managers (otherwise, they will argue through the wall about who is more important). What is the maximum number of managers that can be promoted? Don't forget to explain why this number is indeed maximum. (*I. Tumanova*)
   

7. The chairs on the ski lift are numbered sequentially: 1, 2, 3, etc. The distances between each pair of adjacent chairs are the same. During a thunderstorm, the ski lift stopped, and at that moment chair 22 was at the same height as chair 59, while chair 93 was at the same height as chair 142. Determine the number of chairs on the ski lift. (*L. Koreshkova*)
   

International Mathematical Olympiad
"Formula of Unity" / "The Third Millennium"
Year 2024/2025. Qualifying round

Problems for grade R6

Please hand in your paper in electronic form (e. g. as a doc-file with text or as a scan), some details are at the page formulo.org/en/olymp/2024-math-en/. Your paper should be sent until **23:59:59 UTC, 10 November 2024**.

Please solve the problems by yourself. Remember that the majority of the problems require not only an answer but also its full proof. The paper should not contain your personal data, so **please do not sign your paper**.

---

1. Is it possible to place all numbers from 1 to 8 in a 2×4 rectangle so that the sum of any two numbers which are adjacent by side is a prime number?

   Remark. We remind that a prime number is an integer greater than 1 that is divisible only by 1 and itself. (*M. Karlukova*)

2. In independent work, the teacher gave each of the 10 students a polygon drawn on grid paper along its lines, with an area of 10 cells, and asked them to calculate its perimeter (each student received their own polygon). It turned out that all students had different answers ranging from 13 to 22. What is the minimum number of incorrect answers among them? (*S. Pavlov*)

3. Connor and Mary take turns placing the signs $+$ and $-$ in the squares of an 8×8 board (Connor starts) according to the following rules.

   - Before each move, the player chooses a sign and a color (red, green, or blue) and writes the chosen sign in any free square on the board.
   - If, after a player's move, there is an equal number of pluses and minuses of the chosen color, that player automatically loses.
   - After the board is filled, for each color, it is determined which signs were in greater number. If there are more minuses in one or all three colors, then Mary wins; otherwise, Connor wins.

   Who can ensure their victory, and how should they act to win? (*P. Mulenko*)

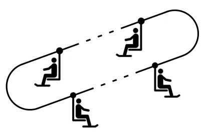
4. Several children came to participate in the final stage of an Olympiad.

   - Among them, there are two girls, each of whom knows exactly four boys.
   - There are three girls, each of whom knows exactly three boys.
   - Each of the remaining girls (if any) knows exactly two boys.
   - No boy knows more than four girls.

   What is the maximum number of girls who could have come to the Olympiad if a total of 15 children participated? Don't forget to prove that this is indeed the maximum number. (*L. Koreshkova*)

5. Consider any three-digit number denoted as $\overline{fdi}$, where each letter represents a separate digit. How many numbers $\overline{fdi}$ exist such that $\overline{idf}$ is divisible by $\overline{fdi}$? (*L. Koreshkova*)

6. The chairs on the ski lift are numbered sequentially: 1, 2, 3, etc. The distances between each pair of adjacent chairs are the same. During a thunderstorm, the ski lift stopped, and at that moment chair 22 was at the same height as chair 59, while chair 93 was at the same height as chair 142. Determine the number of chairs on the ski lift. (*L. Koreshkova*)

7. Sophie has seven friends: Alice, Bella, Dana, Grace, Helena, Jenna, Vicky. Their photos (a total of 7 pictures — one for each friend) are lying in two stacks in an unknown arbitrary order. In one move, Sophie takes several (one or more) consecutive photos from the top of any stack and places them on top of the other stack without changing the order. Is Sophie always able to arrange the photos into one stack with alphabetical order of friends' names (listing from the bottom up) after no more than 13 moves? (*S. Pavlov*)

International Mathematical Olympiad
“Formula of Unity” / “The Third Millennium”
Year 2024/2025. Qualifying round

Problems for grade R7

Please hand in your paper in electronic form (e. g. as a doc-file with text or as a scan), some details are at the page formulo.org/en/olymp/2024-math-en/. Your paper should be sent until **23:59:59 UTC, 10 November 2024**.

Please solve the problems by yourself. Remember that the majority of the problems require not only an answer but also its full proof. The paper should not contain your personal data, so **please do not sign your paper**.

1. Is it possible to place all numbers from 1 to 9 in a 3×3 square so that the sum of any two numbers which are adjacent by side is a prime number? (*M. Karlukova*)
2. Kate drew squares with even sides ranging from 2 cells to 2024 cells in her notebook. Ivy drew a rectangle under each of Kate's squares with the same perimeter but with a width that is 1 less than that of the square. Which girl has a greater total area (in cells) and by how much? (*P. Mulenko*)
3. Find all quadruples of distinct digits $a < b < c < d$ such that $\overline{ab} \cdot \overline{dc} = \overline{ba} \cdot \overline{cd}$.
   Remark. The notation $\overline{ab}$ denotes a two-digit number composed of the digits a and b. (*L. Koreshkova*)
4. Connor and Mary take turns placing the numbers $+1$ and -1 in the squares of an 8×8 board (Connor starts) according to the following rules.
   - Before each move, the player chooses a number and a color (red, green, or blue) and writes the chosen number in any free square on the board.
   - If, after a player's move, the sum of the numbers of the chosen color equals zero, that player automatically loses.
   - After the board is filled, the sums of the numbers of each color are calculated. If the product of these sums is positive, Connor wins; if negative, Mary wins; if any color was never used, it is not included in the count.

   Who can ensure their victory, and how should they act to win? (*P. Mulenko*)
5. Sophie has ten friends: Alice, Chloe, Diana, Fiona, Inna, Jenna, Karina, Lily, Mary, Olivia. Their photos (a total of 10 pictures — one for each friend) are lying in two stacks in an unknown arbitrary order. In one move, Sophie takes several (one or more) consecutive photos from the top of any stack and places them on top of the other stack without changing the order. Is Sophie always able to arrange the photos into one stack with alphabetical order of friends' names (listing from the bottom up) after no more than 21 moves? (*S. Pavlov*)
6. Several children came to participate in the final stage of an Olympiad.
   - Among them, there are two girls, each of whom knows exactly four boys.
   - There are three girls, each of whom knows exactly three boys.
   - Each of the remaining girls (if any) knows exactly two boys.
   - No two children of the same gender know each other.
   - If two children know each other, each of them can give a cheat sheet to the other one.

   It turned out that any girl could send a cheat sheet to any boy (even one she doesn't know, through other children), but if the organizers start closely monitoring at least one pair of children who know each other, this possibility would break down (meaning there would be some boy and girl who would no longer be able to send a cheat sheet to each other). Who came to the Olympiad in a greater number, boys or girls, and by how many? (*L. Koreshkova, P. Mulenko*)
7. Two vending machines sell the same burger, but each of them is broken and changes all the numbers on the screen by some constant value (all other information is correct).

   The company servicing these machines decided to display appropriate notifications during repairs. The following appeared on the screens:

The other machine displays all numbers on the screen 2 more than they actually are. Burger: **\$2**	*The other machine displays all numbers on the screen 8 less than they actually are.* Burger: **\$10**

   What is the real price of the burger? (*P. Mulenko*)

International Mathematical Olympiad
"Formula of Unity" / "The Third Millennium"
Year 2024/2025. Qualifying round

Problems for grade R8

Please hand in your paper in electronic form (e. g. as a doc-file with text or as a scan), some details are at the page formulo.org/en/olymp/2024-math-en/. Your paper should be sent until **23:59:59 UTC, 10 November 2024**.

Please solve the problems by yourself. Remember that the majority of the problems require not only an answer but also its full proof. The paper should not contain your personal data, so **please do not sign your paper**.

---

1. The chess piece "*knishop*" moves one move like a bishop, the next move like a knight, then again like a bishop, and so on (in other words, the moves of bishop and knight alternate). Can it traverse the chessboard visiting all 64 squares exactly once? (*O. Pyayve*)
2. A construction set consists of white cubes. Paul assembles a large cube from all the cubes, then selects 4 faces of the large cube and paints them red. After that, he disassembles the large cube and counts the cubes that have at least one face painted red. Paul ends up with 431 such cubes. Could this happen? If so, find all possible total numbers of cubes. (*L. Koreshkova*)
3. In a trapezoid $ABCD$, the base CD equals 24, $AD = 44$, and the angle B is half of the angle D. What is the maximum possible area of the trapezoid? (*L. Koreshkova, A. Tesler*)
4. Two vending machines sell the same burger, but each of them is broken and changes all the numbers on the screen by multiplying them by some constant value (all other information is correct).

   The company servicing these machines decided to display appropriate notifications during repairs. The following appeared on the screens:

   > *The other machine displays all numbers on the screen* ***100%*** ***more*** *than they actually are.*
   > Burger: **$2**

   > *The other machine displays all numbers on the screen* ***6*** ***times less*** *than they actually are.*
   > Burger: **$12**

   What is the real price of the burger? (*P. Mulenko*)
5. Sophie has N friends with different names: Amelia, Bianca, Eliza, ..., Yana. Their photos (a total of N pictures — one for each friend) are lying in two stacks in an arbitrary order. In one move, Sophie takes several (one or more) consecutive photos from the top of any stack and places them on top of the other stack without changing the order. Is Sophie always able to arrange the photos into one stack with alphabetical order of friends' names (listing from the bottom up) after no more than $2N + 1$ moves? (*S. Pavlov*)
6. Several children came to participate in the final stage of an Olympiad.
   - Among them, there are two girls, each of whom knows exactly four boys.
   - There are three girls, each of whom knows exactly three boys.
   - Each of the remaining girls (if any) knows exactly two boys.
   - No two children of the same gender know each other.
   - If two children know each other, each of them can give a cheat sheet to the other one.

   It turned out that any girl could send a cheat sheet to any boy (even one she doesn't know, through other children), but if the organizers start closely monitoring at least one pair of children who know each other, this possibility would break down (meaning there would be some boy and girl who would no longer be able to send a cheat sheet to each other). Who came to the Olympiad in a greater number, boys or girls, and by how many? (*L. Koreshkova, P. Mulenko*)
7. Alex, Walter, Ruby, and Roxanne are forming natural 5-digit numbers consisting of distinct non-zero digits.
   - Alex writes down all the numbers where the first digit is 1.
   - Walter writes down all the numbers where the first two digits are 1 and 2, in any order.
   - Ruby writes down all the numbers where the first three digits are 1, 2, and 3, in any order.
   - Roxanne writes down all the numbers where the first four digits are 1, 2, 3, and 4, in any order.

   How many five-digit numbers composed of distinct non-zero digits did not appear in any of their lists? (*L. Koreshkova*)

International Mathematical Olympiad
"Formula of Unity" / "The Third Millennium"
Year 2024/2025. Qualifying round

Problems for grade R9

Please hand in your paper in electronic form (e. g. as a doc-file with text or as a scan), some details are at the page formulo.org/en/olymp/2024-math-en/. Your paper should be sent until **23:59:59 UTC, 10 November 2024**.

Please solve the problems by yourself. Remember that the majority of the problems require not only an answer but also its full proof. The paper should not contain your personal data, so **please do not sign your paper**.

---

1. Several kids decided to hold a series of elimination tests, the winner of which would earn the title of the class king. In an interview with the school newspaper, the winner said the following:
   (a) In the first and last tests combined, the same number of participants were eliminated as in all the others combined.
   (b) The number of eliminated participants in the second test was the same as in all subsequent tests in total.
   (c) In the first test, the least number of participants were eliminated.

   Prove that he made a mistake somewhere. (*P. Mulenko*)

2. A construction set consists of white cubes. Paul assembles a large cube from all the cubes, then selects 4 faces of the large cube and paints them red. After that, he disassembles the large cube and counts the cubes that have at least one face painted red. Paul ends up with more than 500 but less than 600 such cubes. How many exactly? Find all possibilities. (*L. Koreshkova*)

3. In a triangle ABC, $\angle A = 135^\circ$, $AB = 14$, $BC = 26$. Point H is the foot of the altitude from point A, and M is the midpoint of AC. Find HM. (*L. Koreshkova*)

4. Solve the equation $[x]^2 + \{x\}^2 = 2x^2$. (Here $[x]$ and $\{x\}$ are the integer and fractional parts of x.) (*S. Pavlov*)

5. A fraction of the form $\frac{1}{n^2}$, where n is a natural number, is called *uni-square*. Find the maximum uni-square fraction that can be expressed as the sum of two uni-square fractions. (*S. Pavlov, A. Tesler*)

6. Several children came to participate in the final stage of an Olympiad.
   - Among them, there are two girls, each of whom knows exactly four boys.
   - There are three girls, each of whom knows exactly three boys.
   - Each of the remaining girls (if any) knows exactly two boys.
   - No two children of the same gender know each other.
   - If two children know each other, each of them can give a cheat sheet to the other one.

   It turned out that any girl could send a cheat sheet to any boy (even one she doesn't know, through other children), but if the organizers start closely monitoring at least one pair of children who know each other, this possibility would break down (meaning there would be some boy and girl who would no longer be able to send a cheat sheet to each other). Who came to the Olympiad in a greater number, boys or girls, and by how many? (*L. Koreshkova, P. Mulenko*)

7. Is it possible to cut a 100×100 square along the cell boundaries into 2024 rectangles so that the union of any set of 2 to 2023 rectangles is not a rectangle? (*A. Tesler*)

International Mathematical Olympiad
"Formula of Unity" / "The Third Millennium"
Year 2024/2025. Qualifying round

Problems for grade R10

Please hand in your paper in electronic form (e. g. as a doc-file with text or as a scan), some details are at the page formulo.org/en/olymp/2024-math-en/. Your paper should be sent until **23:59:59 UTC, 10 November 2024**.

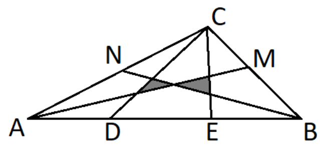
Please solve the problems by yourself. Remember that the majority of the problems require not only an answer but also its full proof. The paper should not contain your personal data, so **please do not sign your paper**.

---

1. In classes 10A and 10B, there are 30 students each. The average height of the boys in class 10A is greater than that of the boys in class 10B. The average height of the girls in class 10A is greater than that of the girls in class 10B. Is it possible that the average height of all students in class 10A is less than that of all students in class 10B? (*A. Tesler*)

2. The function f is defined by the formula $f(x) = \dfrac{2x}{3x^2+1}$. Prove that for any two real mutually inverse numbers s and t, the sum $f(s) + f(t)$ does not exceed 1. (*S. Pavlov*)

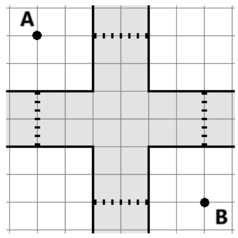
3. In a triangle ABC, two medians AM and BN are drawn. The third vertex C is connected to points D and E, which divide AB into three equal parts. What fraction of the area of the triangle ABC do the two shaded triangles occupy? (*L. Koreshkova*)



4. In a bag for the game Bingo, there are 10 counters with the following numbers: 1, 2, 3, 5, 7, 10, 20, 30, 53, 75. Three counters are taken out of the bag, and the largest number that can be formed by arranging them is recorded. For example, if you take out the counters 7, 20, 30, you record the number 73020. How many numbers greater than 2024 can be recorded? (*L. Koreshkova*)

5. The plan shows an intersection of Horizontal and Vertical streets (the side of each cell is 5 meters, and the crossings are shown as dotted lines). The traffic lights alternate with a period of 2 minutes according to the following schedule:
   - 40 seconds – green light for pedestrians crossing the Horizontal street;
   - the next 20 seconds – red for all pedestrians;
   - the following 40 seconds – green for those crossing the Vertical street;
   - and the last 20 seconds – red for all pedestrians.

   Edgar walks at a speed of 1 m/s. At a random moment, he finds himself at point A, from where he crosses to point B in the fastest way possible, without breaking the rules. Edgar can see how much time is left until the change of signal at each traffic light, so he does not start crossing the street if he cannot finish in time. How many seconds on average will it take Edgar to reach point B? (*A. Tesler*)



6. Segment FI is the diameter of a semicircle which is divided into equal arcs by points $A_1, A_2, \ldots, A_{2n-1}$ (where $n > 2$). A point D is marked on the segment FI, and it turns out that for some k ($1 \leqslant k < \frac{n}{2}$), the sines of the angles A_kDA_{2n-k} and $A_{n-k}DA_{n+k}$ are equal. Prove that D is the midpoint of FI. (*P. Mulenko*)

7. Is it possible to cut a 100×100 square along the cell boundaries into 2024 rectangles so that the union of any set of 2 to 2023 rectangles is not a rectangle? (*A. Tesler*)

International Mathematical Olympiad
"Formula of Unity" / "The Third Millennium"
Year 2024/2025. Qualifying round

Problems for grade R11

Please hand in your paper in electronic form (e. g. as a doc-file with text or as a scan), some details are at the page formulo.org/en/olymp/2024-math-en/. Your paper should be sent until **23:59:59 UTC, 10 November 2024**.

Please solve the problems by yourself. Remember that the majority of the problems require not only an answer but also its full proof. The paper should not contain your personal data, so **please do not sign your paper**.

---

1. Solve the equation $[x]^2 + 2\{x\}^2 = 2x^2$. (Here $[x]$ and $\{x\}$ are the integer and fractional parts of x.) (*S. Pavlov*)

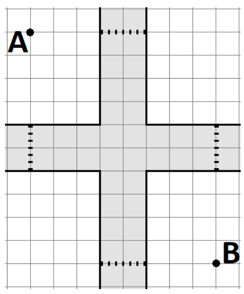
2. Connor and Mary take turns placing the numbers $+1$ and -1 in the squares of an 8×8 board (Connor starts) according to the following rules.
   - Before each move, the player chooses a number and a color (red, green, or blue) and writes the chosen number in any free square on the board.
   - If, after a player's move, the sum of the numbers of the chosen color equals zero, that player automatically loses.
   - After the board is filled, the sums of the numbers of each color are calculated. If the product of these sums is positive, Connor wins; if negative, Mary wins; if any color was never used, it is not included in the count.

   Who can ensure their victory, and how should they act to win? (*P. Mulenko*)

3. A fraction of the form $\frac{1}{n^2}$, where n is a natural number, is called *uni-square*. Find the maximum uni-square fraction that can be expressed as the sum of two uni-square fractions. (*S. Pavlov, A. Tesler*)

4. A line that does not pass through the vertex is drawn through the center of an equilateral triangle with an area of 1. If the triangle is folded along this line, a certain quadrilateral is covered twice. What is the minimum possible area of this quadrilateral? (*L. Koreshkova*)

5. The plan shows an intersection of Horizontal and Vertical streets (the side of each cell is 5 meters, and the crossings are shown as dotted lines). The traffic lights alternate with a period of 2 minutes according to the following schedule:
   - 40 seconds – green light for pedestrians crossing the Horizontal street;
   - the next 20 seconds – red for all pedestrians;
   - the following 40 seconds – green for those crossing the Vertical street;
   - and the last 20 seconds – red for all pedestrians.

   Edgar walks at a speed of 1 m/s. At a random moment, he finds himself at point A, from where he crosses to point B in the fastest way possible, without breaking the rules. Edgar can see how much time is left until the change of signal at each traffic light, so he does not start crossing the street if he cannot finish in time. How many seconds on average will it take Edgar to reach point B? (*A. Tesler*)

6. In the school where Alice studies, marks 1, 2, 3, 4, and 5 are given. Alice received exactly 60 marks in the first quarter. By multiplying them, she obtained a number with digits sum 12. What is the maximum possible arithmetical mean of Alice's marks? (*A. Tesler*)

7. The Tester has a square Polygon. A Source situated in a known point of the Polygon emits 10 types of radiation, which spread along straight and curved paths but cannot cross barriers (each type of radiation has its own type of barrier). The radiation types are divided into good and bad (it is possible that all 10 are good or all 10 are bad), and the Tester does not know which types are good. The optimal life zone is the zone achievable by all types of good radiation but unreachable by the bad one.
   The Tester is preparing for the experiment: he installs 10 barriers (one of each type) so that each barrier divides the Polygon into two parts. After that, he will turn on the Source, and the experiment will begin. The Tester wants the optimal life zone to be connected (i.e., not composed of several separate parts) and its area to be $\frac{1}{1024}$ of the area of the square. Can the Tester install the barriers in such a way to guarantee this? Two barriers can have only a finite number of common points, and a barrier cannot pass through the Source. (*A. Tesler*)